Conformal symmetry and multicritical points in two-dimensional quantum field theory

A. B. Zamolodchikov

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted 11 September 1985)
Yad. Fiz. **44**, 821–827 (September 1986)

It is shown that exactly solvable "minimal" models of two-dimensional conformal quantum field theory describe multicritical points of the scalar field theory with an even polynomial interaction. An analog of this statement is true for the supersymmetric case.

In recent years an effective method has been developed for constructing exact conformally invariant solutions of two-dimensional quantum field theory.[1–4] In principle, these solutions can describe the critical points of certain two-dimensional statistical systems. An important problem which essentially remains unsolved is the determination of the correspondence between exactly solvable models of conformal quantum field theory (CQFT) and phase transitions in physically interesting statistical systems.

A series of "minimal" CQFT models related to highly degenerate representations of the Virasoro algebra was described in Ref. 1. The models of this series are numbered by the integers[1)] $p = 3, 4, 5, \ldots$ (we shall denote them by M_p) and correspond to the values

$$c = 1 - 6/p(p+1) \tag{1}$$

of the central charge of the Virasoro algebra. The models M_3 and M_4 (M_5 and M_6) describe the critical and tricritical behavior Z_2 (Z_3) of the Ising model.[1,5,6] There is an analogous series of "minimal" supersymmetric models SM_p, $p = 3, 4, 5, \ldots$, related to representations of the Neveu-Schwarz-Ramond algebra and corresponding to the values of the central charge

$$c = {}^3/_2\,(1 - 8/p(p+2)) \tag{2}$$

(Refs. 3, 7, and 8). It is known that $SM_3 = M_4$ (Ref. 3), that SM_4 describes a special case of the Ashkin-Teller critical model, and that SM_6 describes the critical point Z_6 of the Ising model.[4]

In the present study we present arguments supporting the view that the models of the series M_p describe the multicritical points of the two-dimensional scalar field theory with even (i.e., invariant under the replacement $\varphi \to -\varphi$) polynomial interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int} = \int d^2x \left[\frac{1}{2}\,\partial_\mu\varphi\,\partial_\mu\varphi + V_N(\varphi^2) \right],$$

$$V_N = \sum_{k=1}^{N} g_k \varphi^{2k}. \tag{3}$$

An analogous statement can be made for the models SM_p, as will be shown below.

For a given N, in the N-dimensional space of the parameters g_k there exist hypersurfaces S_l, $l = 2, 3, 4, \ldots, N$ of codimension $l - 2$ (i.e., dimension $N + 2 - l$), along which l phases can coexist in the model (3). At points on the hypersurface S_l the effective potential $V_{\rm eff}(\varphi)$ of the model (3) has l degenerate minima. An example of such a potential is shown in Fig. 1. The $(N + 1 - l)$-dimensional boundary (rather, one of the components of the boundary) C_l of the hypersurface S_l is critical.[2)] Whereas the form of the actual hypersurfaces S_l and C_l has an essential dependence on the regularization scheme of the theory (3),[1)] the critical behavior at the hypersurface C_l is universal and depends only on l; we shall refer to it as the l-critical behavior of the model (3). In this terminology the 2-critical behavior corresponds to the Ising phase transition.[9] Our statement is that the $(p - 1)$-critical behavior of the theory (3) is described by the "minimal" model M_p. We note that the $(p - 1)$-critical point corresponds to the vanishing of the first $2p - 3$ derivatives of $V_{\rm eff}$ at $\varphi = 0$, so that such a point can be described by the effective interaction

$$\mathcal{L}_{\rm int} = g \int \varphi^{2(p-1)}\, d^2x. \tag{4}$$

We shall justify the above statement by comparing the structures of the operator algebras in the critical theory (4) and the model M_p. The conformally invariant operator algebra[1] describing the $(p - 1)$-critical theory (4) contains, in addition to the "fundamental" field φ, all possible renormalized composite fields of the form: $:\varphi^k:$, $:\varphi^k\,\partial_\mu\varphi\,\partial_\mu\varphi:$, and so on with different (universal for each surface C_{p-1}) anomalous dimensions. A few remarks should be made about the definition of the composite fields. Let us consider, for example, the operator expansion of the product $\varphi(x)\varphi(0)$, which has the form

$$\varphi(x)\varphi(0) = \langle\varphi(x)\varphi(0)\rangle + |x|^{d_2 - 2d_1}\varphi_2(0) + \ldots, \tag{5}$$

where on the right-hand side we have written out only the term most singular for $x \to 0$; the parameters d_1 and d_2 are the the anomalous dimensions[4)] of the fields φ and φ_2, respectively. The relation (5) serves as the definition of the com-

FIG. 1. Example of the form of the effective potential for the hypersurface S_l ($l = 5$).

 0038-5508/86/090529-05$03.80

posite field $:\varphi^2:\equiv\varphi_2$. The composite fields $:\varphi^{k+1}:$ can be defined recursively as the corresponding singular terms in the operator expansions of $\varphi(x):\varphi^k:(0)$. Here it should be remembered that the most singular (for $x\to 0$) term in this expansion comes from the fields $:\varphi^{k-2q}:$, $q=1,2,\ldots,\leqslant k/2$; these terms must be subtracted. Therefore,

$$:\varphi^{k+1}:(0)=\lim_{x\to 0}|x|^{d_k+d_1-d_{k+1}}\Big\{\varphi(x):\varphi^k:(0)-\sum_{q=1}^{[k/2]}A_q \times|x|^{d_{k-2q}-d_1-d_k}:\varphi^{k-2q}:(0)\Big\}, \qquad (6)$$

where d_q with $q\leqslant k$ is the dimension of the fields $:\varphi^q:$ and the coefficients A_q and the exponent d_{k+1} are chosen so as to ensure the existence of a finite limit in (6); here d_{k+1} coincides with the anomalous dimension of the field $:\varphi^{k+1}:$. When necessary, it is understood that in (6) there is averaging over the directions of the vector x_μ in order that vector terms do not appear. Other composite fields, for example, $:\varphi^k\partial_\mu\varphi\partial_\nu\varphi:$ and so on, can also be defined in a similar manner.

In the $(p-1)$-critical theory described by the interaction (4) the composite field $:\varphi^{2p-3}:$ constructed using (6) must coincide up to a numerical coefficient with the second derivative of the field φ (the first derivatives drop out upon averaging over directions of x in (6)), that is,

$$g:\varphi^{2p-3}:=\partial_\mu\partial_\mu\varphi. \qquad (7)$$

This relation is simply the operator equation of motion for the theory (4).

The operator algebra of the model M_p contains $p(p-1)/2$ scalar conformal fields $\phi_{(n,m)}=\phi_{(p+1-n,p-m)}$, $n=1,2,\ldots,p$, $m=1,2,\ldots,p-1$, with anomalous dimensions[1]

$$d_{(n,m)}=\frac{(pn-(p+1)m)^2-1}{2p(p+1)}, \qquad (8)$$

where the field $\phi_{(1,1)}=\phi_{(p,p-1)}=I$ is the unit operator. It is convenient to write the field $\phi_{(n,m)}$ in the form of a rectangular table, an example of which is shown in Fig. 2. The structure of the operator algebra of the model M_p is described in Ref. 1 and in more detail in Ref. 11. We need the relations

$$\phi_{(2,2)}\phi_{(n,m)}=\sum_{k,l=\pm 1}C^{(k,l)}_{n,m}[\phi_{(n+k,m+l)}+\ldots], \qquad (9)$$

where we have omitted the arguments of the fields ϕ and the usual power factors on the right-hand side. The square brackets in (9) denote the contribution of all the fields belonging to the "conformal class" of the field $\phi_{(n+k,m+l)}$ (Ref. 1) and the $C^{(k,l)}_{n,m}$ are the structure constants of the operator algebra. These quantities are calculated for the models M_p in Ref. 11. For us it is important only that

$$C^{(-,-)}_{n,1}=C^{(+,-)}_{n,1}=C^{(-,+)}_{n,p-1}=C^{(+,+)}_{n,p-1}=C^{(-,-)}_{1,m}=C^{(-,+)}_{1,m}=0. \qquad (10)$$

If we use the notation $\phi_{(2,2)}=\varphi$ and define "composite" fields $:\varphi^k:$ using the rules (6), we easily verify using (9), (10), and (8) that

$$:\varphi^k:\approx\phi_{(k+1,k+1)} \quad \text{for} \quad k=0,1,\ldots,p-2, \qquad (11)$$
$$:\varphi^k:\approx\phi_{(k-p+2,k-p+3)} \quad \text{for} \quad k=p-1,p,p+1,\ldots,2p-4.$$

The most singular contribution to the product $\varphi:\varphi^{2p-4}:=\phi_{(2,2)}\phi_{(p-2,p-1)}$ comes, according to (9), from the fields $\phi_{(p-3,p-2)}=:\varphi^{2p-5}:$ and $\phi_{(p-1,p-2)}=\phi_{(2,2)}=\varphi$. In accordance with (6), these terms should be subtracted in the definition of the field $:\varphi^{2p-3}:$. Therefore, (6) will receive a contribution from the field $\partial_\mu\partial_\mu\varphi$, which is the most singular (after the field φ itself) scalar representative of the conformal class $[\varphi]$.[5] Therefore, equation (7) is valid for the field $:\varphi^{2p-3}:$, that is, the field $\varphi=\phi_{(2,2)}$ in the model M_p formally satisfies the operator equation of motion of the model (4), which proves our statement. According to (11) and (8), the dimensions of the composite fields $:\varphi^k:$ in the $(p-1)$-critical theory (4) are

$$d_k=\begin{cases}\dfrac{(k+1)^2-1}{2p(p+1)}, & k=0,1,2,\ldots,p-2\\[2mm] \dfrac{(k+3)^2-1}{2p(p+1)}, & k=p-1,p,\ldots,2p-4\end{cases}. \qquad (12)$$

The other fields $\phi_{(n,m)}$ which are not involved in (11) correspond to scalar composite fields of the type $:\varphi^k\partial_\mu\varphi\partial_\mu\varphi:$ For example, it is natural to identify the operator $\phi_{(1,3)}$ with the field $:\partial_\mu\varphi\partial_\mu\varphi:$, which therefore has dimension

$$d=2(1+2/p). \qquad (13)$$

The technique developed in Refs. 1 and 11 also allows the calculation of multi-point correlation functions of the fields $:\varphi^k:$. For example, the four-point function $\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle$ can be expressed in terms of hypergeometric functions. Since this result has not been published before, we give the corresponding formula in the Appendix.

It should be noted that the conclusion arrived at above is not entirely unexpected. It was shown in Ref. 12 that the models M_p describe the "ferromagnetic" critical points of the exactly solvable "RSOS model" (Ref. 13), which apparently have the same physical origin as the $(p-1)$-critical points of the theory (3). Our principal conclusion is that a critical theory M_p is not a specific feature of some exactly solvable model, but describes the general $(p-1)$-critical behavior of two-dimensional systems with scalar order parameter φ and symmetry $Z_2(\varphi\to-\varphi)$.

Similar arguments show that the models SM_p describe $(p-1)$-critical points of supersymmetric field theory:

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_{\rm int}=\int d^2x\,d^2\theta\left[\frac{1}{2}D\Phi\,\bar{D}\Phi+W_N(\Phi)\right], \qquad (14)$$

where

$$D=\partial_\theta-\bar{\theta}\partial_z,\quad \bar{D}=\partial_{\bar\theta}-\theta\partial_{\bar z},\quad z=x^1+ix^2,\quad \bar z=x^1-ix^2,$$

θ and $\bar\theta$ are the complex coordinates of $2+2$-dimensional superspace, $\Phi(x,\theta,\bar\theta)$ is the superfield

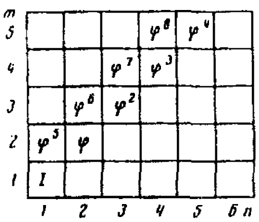

FIG. 2. Table of the conformal fields $\Phi_{(n,m)}$ for the model M_p, $p = 6$, corresponding to the interaction φ^{16}.

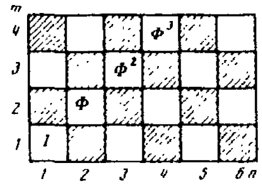

FIG. 3. Table of the conformal superfields $\Phi_{(n,m)}$ for SM_p, $p = 5$ (the interaction Φ^5). The cells corresponding to Ramond states $\sigma_{(n,m)}$ are shaded.

$$\Phi=\varphi+\theta\psi+\bar\theta\bar\psi+\theta\bar\theta\chi, \tag{15}$$

and

$$W_N(\Phi)=\sum_{k=1}^{N} g_k\Phi^k.$$

The free Lagrangian in (14) is invariant under two reflections:

$$(\theta,\ \bar\theta)\to(\theta,\ \bar\theta),\qquad \Phi\to-\Phi, \tag{16a}$$
$$(\theta,\ \bar\theta)\to(\theta,\ -\bar\theta),\qquad \Phi\to-\Phi. \tag{16b}$$

The theory (14) will therefore be invariant under (16a) if $W_N(-\Phi) = W_N(\Phi)$ and under (16b) if $W_N(-\Phi) = -W_N(\Phi)$. We shall assume that for even (odd) N the potential W_N contains only even (odd) powers of Φ. As in (4), a $(p-1)$-critical point of the model (14) can be described by the effective potential

$$\int W_{\rm eff}(\Phi)\,d^2x\,d^2\theta = g\int d^2x\,d^2\theta\,\Phi^p(x,\theta,\bar\theta). \tag{17}$$

The model SM_p contains $[p^2/2]$ (square brackets denote the integer part) conformal superfields

$$\Phi_{(n,m)}=\Phi_{(p+2-n,\ p-m)},\quad n=1,\ 2,\ \dots,\ p+1,\ m=1,\ 2,\ \dots,\ p-1$$

($n+m$ odd) with dimension[5]

$$d_{(n,m)}=[(np-m(p+2))^2-4]/4p(p+2). \tag{18}$$

It is convenient to expand these fields in a table of dimension $(p-1)\times(p+1)$ as in Fig. 2; an example is shown in Fig. 3. The superfields $\Phi_{(n,m)}$ form a closed operator algebra,[7,8] with relations of the type (9) valid also for SM_p if the substitution $\phi_{(n,m)}\to\Phi_{(n,m)}$ is made. Making the identification $\Phi_{(2,2)} = \Phi$, we can verify that

$$:\Phi^k:\approx\Phi_{(k+1,\ k+1)},\qquad k=0,\ 1,\ 2,\ \dots,\ p-2, \tag{19}$$

and that the superfields Φ, like (7), satisfies the operator equation of motion

$$D\bar D\Phi=g{:}\Phi^{p-1}{:} \tag{20}$$

of the theory (17). Therefore, the dimensions d_k of the composite superfields $:\Phi^k:$ at a $(p-1)$-critical point of the model (14) are equal to[6]

$$d_k=[(k+1)^2-1]/p(p+2) \tag{21}$$

We note that the model SM_p contains, in addition to the superfields $\Phi_{(n,m)}$, the conformal fields $\sigma_{(n,m)}$, $n = 1, 2, \dots, p+1$; $m = 1, 2, \dots, p-1$ ($n+m$ odd). These "spin" fields correspond to Ramond representations of the algebra of superconformal transformations[5] and cannot be expressed in a local form in terms of the superfield Φ.

Let us make a few remarks about the possible physical interpretation of phase transitions in the model (14). First let us consider a statistical system in which an Ising "spin" variable $\sigma = \pm 1$ interacts locally with some other degrees of freedom, which we refer to as the "heat reservoir or bath". The interaction is invariant under the replacement $\sigma\to-\sigma$. Let the state of the heat bath be described by a scalar local order parameter $\varphi(x)$ and the local energy density of the bath be $U(\varphi)$. Then a Ginzburg-Landau type of Hamiltonian for this system can be written as $\mathcal{H} = \mathcal{L}_0 + \mathcal{L}_1$, where $\mathcal{L}_0$ has the same form as (14) and

$$\mathcal{L}_1=\int d^2x\,[U(\varphi)\bar\psi\psi+V(\varphi)]. \tag{22}$$

Here ψ and $\bar\psi$ are the components of a Majorana Fermi field related in the standard manner to the Ising spins σ (Ref. 9). Of course, the question of to what degree a phase transition will be supersymmetric must be answered separately for each system. In any case, it is instructive that the tricritical behavior of the Ising spins is supersymmetric.[5]

In conclusion, we stress the fact that the arguments given above do not, of course, provide a rigorous proof of our statements. It is therefore useful to verify our conclusions using the ε-expansion technique, for example.[10] The models (4) and (17) can be studied in a space of arbitrary dimension $\mathcal{D}$ with the upper critical dimension in both cases being $\mathcal{D}_p^* = 2(p-1)/(p-2)$. Setting $\mathcal{D} = \mathcal{D}_p^* - \varepsilon$, exponents describing the $(p-1)$-critical points of the models (3) and (14) can be sought in the form of a series in powers of ε. The known results for the model (4) with $p = 3$ and 4 (Refs. 10 and 14) are, of course, in agreement with our statement. For (4) with $p > 4$ and for (17) the corresponding calculations have apparently not been carried out (in relation to this see Ref. 15).

APPENDIX

Introducing the complex coordinates

$$z_j=x_j^1+ix_j^2,\quad \bar z_j=x_j^1-ix_j^2. \tag{A1}$$

the four-point function $\langle\varphi(x_1)\dots\varphi(x_4)\rangle$ (and every conformal four-point function) can be written as

$$\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle=(z_{13}\bar z_{13}z_{24}\bar z_{24})^{-\Delta}G(x,\bar x), \quad (A2)$$

where

$$z_{ij}=z_i-z_j,\ \bar z_{ij}=\bar z_i-\bar z_j;$$

x and $\bar x$ are projective invariants

$$x=z_{12}z_{34}/z_{13}z_{24},\ \bar x=\bar z_{12}\bar z_{34}/\bar z_{13}\bar z_{24}. \quad (A3)$$

For the $(p-1)$-critical theory (4) (that is, for the model M_p) the function $G(x,\bar x)$ can be written as

$$G(x,\bar x)=\sum_{a,b=1,3}C^2_{(a,b)}(p)\mathcal{F}_{(a,b)}(p|x)\mathcal{F}_{(a,b)}(p|\bar x), \quad (A4)$$

where the functions $\mathcal{F}_{a,b}$ (the "conformal blocks", Ref. 1) have the form

$$\mathcal{F}_{(1,1)}(p|x)$$
$$=[x(1-x)]^{-\Delta_1}\Big\{2(1-x)F\Big(\frac{1}{p+1},\frac{3}{p+1},\frac{2}{p+1},x\Big)$$
$$\times F\Big(1-\frac{3}{p},1-\frac{1}{p},1-\frac{2}{p},x\Big)$$
$$+\frac{x}{p-2}F\Big(\frac{1}{p+1},\frac{3}{p+1},1+\frac{2}{p+1},x\Big)$$
$$\times F\Big(1-\frac{3}{p},1-\frac{1}{p},2-\frac{2}{p},x\Big)\Big\}. \quad (A5a)$$

$$\mathcal{F}_{(1,3)}(p|x)=[x(1-x)]^{-\Delta_1}x^{2/p}$$
$$\Big\{(1-x)F\Big(\frac{1}{p+1},\frac{3}{p-1},\frac{2}{p+1},x\Big)$$
$$\times F\Big(1+\frac{1}{p},1-\frac{1}{p},1+\frac{2}{p},x\Big)$$
$$-F\Big(\frac{1}{p+1},\frac{3}{p-1},1+\frac{2}{p+1},x\Big)$$
$$\times F\Big(\frac{1}{p},-\frac{1}{p},\frac{2}{p},x\Big)\Big\}. \quad (A5b)$$

$$\mathcal{F}_{(3,1)}(p|x)=[x(1-x)]^{-\Delta_1}x^{1-2/(p+1)}$$
$$\times\Big\{-\frac{1-x}{p+1}F\Big(1+\frac{1}{p+1},1-\frac{1}{p+1},2-\frac{2}{p+1},x\Big)$$
$$\times F\Big(1-\frac{3}{p},1-\frac{1}{p},1-\frac{2}{p},x\Big)$$
$$+\frac{1}{p-2}F\Big(\frac{1}{p+1},-\frac{1}{p+1},1-\frac{2}{p+1},x\Big)$$
$$\times F\Big(1-\frac{3}{p},1-\frac{1}{p},2-\frac{2}{p},x\Big)\Big\}, \quad (A5c)$$

$$\mathcal{F}_{(3,3)}(p|x)=[x(1-x)]^{-\Delta_1}x^{4/p}\Big\{\frac{x(1-x)}{p-1}$$
$$\times F\Big(1+\frac{1}{p+1},1-\frac{1}{p+1},2-\frac{2}{p+1},x\Big)$$
$$\times F\Big(1+\frac{1}{p},1-\frac{1}{p},1+\frac{2}{p},x\Big)$$
$$+2F\Big(\frac{1}{p+1},-\frac{1}{p+1},1-\frac{2}{p+1},x\Big)$$
$$\times F\Big(\frac{1}{p},-\frac{1}{p},\frac{2}{p},x\Big)\Big\}. \quad (A5d)$$

and $F(a,b,c,x)$ is a hypergeometric function. The four terms in (A4) respectively describe the contribution of the composite fields (or rather, the contribution of the conformal classes[1])

$$I=\phi_{(1,1)},\ :\partial_\mu\varphi\partial_\mu\varphi:\approx\phi_{(1,3)},\ :\varphi^{2p-1}:\approx\phi_{(3,1)}\text{ and }:\varphi^2:=\phi_{(3,3)}$$

to the operator expansion of $\varphi(x_1)\varphi(x_2)$. The coefficients $C_{(a,b)}(p)$ in (A4) are selected such that the cross-invariance of the correlation function (A2) is ensured; assuming that the field $\varphi(x)$ is normalized such that

$$\langle\varphi(x)\varphi(0)\rangle=(z\bar z)^{-\Delta_1}, \quad (A6)$$

they have the form

$$C^2_{(1,1)}=1,$$
$$C^2_{(1,3)}=\frac{3}{4}\frac{1}{(p+2)^2(p+3)^2}\frac{\Gamma(1+1/p)\Gamma^2(1-2/p)\Gamma(1+3/p)}{\Gamma(1-1/p)\Gamma^2(1+2/p)\Gamma(1-3/p)},$$
$$C^2_{(3,1)}=\frac{3}{4}\frac{1}{(p-1)^2(p-2)^2}\frac{\Gamma(1-1/(p+1))\Gamma^2(1+2/(p+1))\Gamma(1-3/(p+1))}{\Gamma(1+1/(p+1))\Gamma^2(1-2/(p+1))\Gamma(1+3/(p+1))}$$
$$C^2_{(3,3)}=\frac{\Gamma(1+1/p)\Gamma^2(1-2/p)\Gamma(1+3/p)}{\Gamma(1-1/p)\Gamma^2(1+2/p)\Gamma(1-3/p)}$$
$$\times\frac{\Gamma(1-1/(p+1))\Gamma^2(1+2/(p+1))\Gamma(1-3/(p+1))}{\Gamma(1+1/(p+1))\Gamma^2(1-2/(p+1))\Gamma(1+3/(p+1))} \quad (A7)$$

(all the structure constants for the models M_p have been calculated in Ref. 11). We note that $C_{(3,1)} \to 0$ and $C_{(1,3)} \to 0$ if $p \to 0$; this corresponds to the interaction being weak at large p.

[1)] Actually, in Ref. 1 a set of "minimal" models corresponding to any rational value $p > 0$ was constructed. It was shown in Ref. 2, however, that only the integer-valued series (1) satisfies the condition of unitarity, which must hold for systems with a Hamiltonian that is real and bounded below. The other "minimal" models might describe complex critical points (see Ref. 5).

[2)] That is, the correlation radius at the points C_l diverges.

[3)] Of course, the values of the coefficients g_k of the polynomial (3) are meaningful only within the context of a regularization scheme and when the composite fields φ^{2k} are defined.

[4)] Here we have used the standard definition of the anomalous dimensions[10] corresponding to the two-point function $\langle\varphi(x)\varphi(0)\rangle \sim |x|^{-2d_i}$.

[5)] The structure of the "conformal classes" and their relation to representations of the Virasoro algebra are described in detail in Ref. 1.

[6)] As usual, by the dimension $\tilde{d}$ of a superfield Φ we mean the dimension of its lowest bosonic componet. The fermion components have dimension $\tilde{d} + 1/2$.

[1] A. A. Belavin, A. M. Polyakov, and A. B. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984).

[2] D. Friedan, Z. Qiu, and S. Shenker, Phys. Lett. **52**, 1575 (1984).

[3] D. Friedan, Z. Qiu, and S. Shenker, Phys. Lett. **151B**, 37 (1985).

[4] A. B. Zamolodchikov and V. A. Fateev, Zh. Eksp. Teor. Fiz. 89, 380 (1985) [Sov. Phys. JETP **62**, 215 (1985)].

[5] J. L. Cardy, Phys. Rev. Lett. **54**, 1354 (1985).

[6] V. S. Dotsenko, Nucl. Phys. **B235** [FS 11], 54 (1984).

[7] H. Eichenherr, Phys. Lett. **151B**, 26 (1985).

[8] M. A. Bershadsky, V. G. Knizhnik, and M. G. Teitelman, Phys. Lett. **151B**, 31 (1985).

[9] B. M. NcCoy and T. T. Wu, *The Two-Dimensional Ising Model*, Harvard University Press, Cambridge, Mass., 1973.

[10] K. Wilson and J. Kogut, *The Renormalization Group and ε Expansion*, Phys. Lett. C12, 75 (1974) [Russ. transl., Mir, 1975].

[11] V. S. Dotsenko and V. A. Fateev, Nucl. Phys. **240** [FS 12], 312 (1984).

[12] D. A. Huse, Phys. Rev. B **30**, 3908 (1984).

[13] G. E. Andrews, R. J. Baxter, and P. J. Forrester, J. Stat. Phys. 35, 193 (1984).

[14] M. den Nijs, Phys. Rev. B **27**, 1674 (1983).

[15] L. N. Lipatov, Zh. Eksp. Teor. Fiz. **64**, 551 (1975) (*sic*).

Translated by Patricia Millard